12.

(a) $\pi = \underbrace{PQ}_{\text{Revenue}} - \underbrace{C}_{\text{total cost.}}$

Revenue $P \cdot Q = \left(1000 - \left(\frac{1}{3}\right)Q\right) Q = 1000Q - \frac{1}{3}Q^2$

$C = \underbrace{\left(800 + \frac{1}{5}Q\right)}_{\text{Input price}} \underbrace{Q}_{\text{Input quantity}} + \underbrace{100Q}_{\text{transportation cost}}$

$\pi = 1000Q - \frac{1}{3}Q^2 - 800Q - \frac{1}{5}Q^2 - 100Q$

$= 100Q - \frac{8}{15}Q^2$

$\begin{cases} a = -\frac{8}{15} \\ b = 100 \\ c = 0 \end{cases}$ Since $a < 0$, π is maximized at

$Q^* = \frac{-b}{2a} = \frac{-100}{2\left(-\frac{8}{15}\right)} = \frac{(100)(15)}{16} = 93.75.$

(b) Now cost increase by $10Q$

$\pi = 1000Q - \frac{1}{3}Q^2 - 800Q - \frac{1}{5}Q^2 - 100Q - 10Q$

$= 90Q - \frac{8}{15}Q^2$

$Q^* = -\frac{b}{2a} = -\frac{90}{2\left(-\frac{8}{15}\right)} = \frac{90(15)}{16} = 84,375$