ECO13F Practice Questions for chapter 13. "Multivariable Optimization"

Sec. 13.1

2. (a) The function f defined for all (x, y) by $f(x,y) = x^2 + y^2 - 6x + 8y + 35$ has a minimum point. Find it

3. Find the values of K & L that maximize the profit defined as below,

$$\Pi = pF(K,L) - rK - wL$$

$$F(K,L) = 80 - (K-3)^2 - 2(L-6)^2 - (K-3)(L-6)$$

$$p = 1,\ r = 0.65,\ w = 1.2.$$

4. Yearly profits for a firm are given by

$$P(x,y) = -x^2 - y^2 + 22x + 18y - 102$$

where x is the amount spent on research ~~for~~ and y is the amount spent on advertising

(a) Find the profits when $x = 10,\ y = 8$ & when $x = 12,\ y = 10$.

(b) Find x & y that maximize profits. Find maximized profit.

Sec. 13.2.

2.(a) A firm produces two different kinds A and B of a commodity. The daily cost of producing x units of A and y units of B is

$$C(x,y) = 2x^2 - 4xy + 4y^2 - 40x - 20y + 514.$$

Suppose that the firm sells all its output at a price per unit of \$24 for A and \$12 for B. Find the daily production levels x and y that maximizes profit.

(b) The firm is required to produce exactly 54 units per day of the two kinds combined. What now is the optimal production plan?

4. The demands for a monopolist's two products are determined by the equations

$$p = 25 - x,\quad q = 24 - 2y$$

Where p and q are prices per unit of the two goods, and x and y are the corresponding quantities. The cost of producing & selling x units of the first good and y units of the other are $C(x,y) = 3x^2 + 3xy + y^2$

Find the values of x and y that maximizes $\pi(x,y)$. Verify that it's maximum.

6. The profit function of a firm is $\pi(x,y) = px + qy - \alpha x^2 - \beta y^2$,
where p and q are the prices per unit and $\alpha x^2 + \beta y^2$ are the costs of producing & selling x units of the 1st good & y units of the other.
The constants are all positive.

(a) Find the values of x and y that maximizes profit.
Verify S.O.C.

Sec. 13.3.

1. Consider the function f defined for all (x, y) by
$f(x,y) = 5 - x^2 + 6x - 2y^2 + 8y$.

(a) Find all the partial derivatives of first & second order.

(b) Find the only stationary point and classify it by using S.O.C.

Sec. 13.4

1. (a) Suppose the demand functions of a monopolist in 2 markets are

$$\begin{cases} P_1 = 200 - 2Q_1 \\ P_2 = 180 - 4Q_2 \end{cases}$$ and the cost function is

$$C = 20(Q_1 + Q_2).$$

Derive Q_1^*, Q_2^*, P_1^*, P_2^* & π^*.

(b) How much profit is lost if it becomes illegal to price discriminate?

(c) Discuss the consequences of imposing a tax of 5 per unit sold in market 1.

Sec. 13.5.

1. Let $f(x,y) = 4x - 2x^2 - 2y^2$, $S = \{(x,y) : x^2 + y^2 \leq 25\}$.

(a) Compute $f_1'(x,y)$ and $f_2'(x,y)$, then find the only stationary point for f.

(b) Find the extreme points for f over S.

3. Find x^*, y^* that maximizes

$$f(x,y) = 9x + 8y - 6(x+y)^2$$, subject to $$\begin{cases} 0 \leq x \leq 5 \\ 0 \leq y \leq 3 \\ -x + 2y \leq 2 \end{cases}$$

Sec. 13.6.

1. Find a maximum point for $f(x,y,z) = 3 - x^2 - 2y^2 - 3z^2 - 2xy - 2xz$.

Review problems

2. A firm produces two different kinds A and B of a commodity. The daily cost of producing Q_1 units of A and Q_2 units of B is $C(Q_1, Q_2) = 0.1(Q_1^2 + Q_1Q_2 + Q_2^2)$. Suppose that the firm sells all its output at a price per unit of

$P_1 = 120$ for A and $P_2 = 90$ for B.

(a) Find the daily production levels that maximizes profits.

(b) What price (P_1) per unit of A would imply that the optimal daily production level for A is 400 units?

3. (a) The profit obtained by a firm from producing and selling x and y units of two brands of a commodity is given by

$$P(x, y) = -0.1x^2 - 0.2xy - 0.2y^2 + 47x + 48y - 600.$$

Find the production levels that maximize profits.

(b) When the total production level is set to 200 units, find the production levels that maximizes profit.

5. (a) Suppose the production function $Q = a\ln K + b\ln L + c\ln T$. and profit function is

$$\pi = p(a\ln K + b\ln L + c\ln T) - rK - wL - gT.$$

Find K^*, L^*, T^* that maximize profit.

(b) Derive $\dfrac{\partial \pi^*}{\partial r}$.

7. (a) Find and classify the stationary points of

$$f(x, y) = x^2 - y^2 - xy - x^3.$$