ECO138: Practice Questions for Chapter 14

Section 14.1.

1. Use Lagrange's method to find the only possible solution to the problem:

max xy subject to x + 3y = 23.

2. Use the Lagrange's method to solve the problem

min $-40Q_1 + Q_1^2 - 2Q_1Q_2 - 20Q_2 + Q_2^2$ subject to $Q_1 + Q_2 = 15$

3. Solve the following problems.

(a) max $10x^{1/2}y^{1/3}$ $\quad subject\ to\ 2x + 4y = m$

(b) max $x^{1/2}y^{1/2}$ $\quad subject\ to\ 50000x + 0.08y = 1000000$

(c) max $12x\sqrt{y}$ $\quad subject\ to\ 3x + 4y = 12$

4. (a) min $f(x,y) = x^2 + y^2 \quad subject\ to\ g(x,y) = x + 2y = 4$

(b) min $f(x,y) = x^2 + 2y^2 \quad subject\ to\ g(x,y) = x + y = 12$

(c) min $f(x,y) = x^2 + 3xy + y^2 \quad subject\ to\ g(x,y) = x + y = 100$

5. A person has utility function

$$u(x,y) = 100xy + x + 2y$$

Suppose that the price per unit of x is \$2, and that the price per unit of y is \$4. The person receives \$1000 that all has to be spent on the two commodities x and y. Solve the utility maximization problem.

8. A firm produces and sells two commodities. By selling x tons of the first commodity the firm gets a price per ton given by p = 96 - 4x. By selling y tons of the other commodity the price per ton is given by q = 84 -2y. The cost of producing and selling x tons of the first commodity and y tons of the second is given by $C(x,y) = 2x^2 + 2xy + y^2$. Compute the first order partial derivatives of P, and find its only stationary point.

Section 14.2

2. Solve the problem $minimixe\ rK + xL\ subject\ to\ \sqrt{K} + L = Q, assuming\ that\ Q > \frac{w}{2r}, where\ r, w\ and\ Q\ are\ positive\ constants.$

4. (a) Solve the utility maximization problem

$$\max U(x, y) = \sqrt{x} + y, subject\ to\ x + 4y = 100$$

using the Lagrange method, i.e. find the quantities demanded of the two goods.

(b) Suppose income increases from 100 to 101. What is the exact increase in the optimal value of U(x,y)? Compare with the value found in (a) for the Lagrange multiplier.

(c) Suppose we change the budget constraint to px + qy = m, but keep the same utility function. Derive the quantities demanded of the two goods if $m > \frac{q^2}{4p}$.

Section 14.3.

1. (a) max(min) 3xy subject to $x^2 + y^2 = 8$.

(b) max(min) x + y subject to $x^2 + 3xy + 3y^2 = 3$.

4. Solve the problem

max $f(x, y) = 24x - x^2 + 16y - 2y^2$ subject to $g(x, y) = x^2 + 2y^2 = 44$

What is the approximate change in the optimal value of f(x,y) if 44 is changed to 45?

Section 14.5

2. Consider the utility maximizing problem max lnx + lny subject to px + qy = m.

Section 14.6

1. Consider the problem min $x^2 + y^2 + z^2\ subject\ to\ x + y + z = 1$.

Write down the Lagrangian for this problem, and find the only point (x,y,z) that satisfies the necessary conditions.

2. Solve the utility maximizing problem

$$\max\ \ 10\, x^{1/2} y^{1/3} z^{1/4}\ \ subject\ to\ 4x + 3y + 6z = 390$$

6. Solve the problem:

max(min) x + y subject to $\begin{cases} x^2 + 2y^2 + z^2 = 1 \\ x + y + z = 1 \end{cases}$

Section 14.7

1. (a) Assuming $0 \leq a < \frac{m}{p}$, find the solution (x*, y*) to the utility maximization problem

$$\max x + alny \quad subject\ to\ px + qy = m$$

(b) Find the indirect utility function U*(p,q,m,a) = x* + lny*, and compute its partial derivatives with respect to p, q, m and a. Verity the envelope theorem.

2. Solve the problem min $x + 4y + 3z \quad subject\ to\ x^2 + 2y^2 + \frac{1}{3}z^2 = b$. (Suppose that b >0 and take it for granted that the problem has a solution.)

4. (a) Solve the problem

$$\max(\min) f(x,y,z) = x^2 + y^2 + z \quad subject\ to\ g(x,y,z) = x^2 + 2y^2 + 4z^2 = 1.$$

(b) Suppose the constraint is changed to $x^2 + 2y^2 + 4z^2 = 1.02$. What is the approximate change in the maximum value of f(x,y,z)?

5. Solve min C = rK + wL subject to F(K,L) = $K^{1/2}L^{1/4}$. Find explicit expressions for K*, L*, C* and λ.

Section 14.8

2. (a) Write down the Kuhn Tucker conditions for the problem

$\max x^2 + 2y^2 - x \quad subject\ to\ x^2 + y^2 \leq 1.$

(b) Find all pairs (x, y) that satisfy all the necessary conditions. (There are five candidates.) Find the solution to the problem.

3. Consider the problem

$\max f(x,y) = 2 - (x-1)^2 - e^{y^2} \quad subject\ to\ x^2 + y^2 \leq a$, where a is a positive constant.

(a) Write down the Kuhn-Tucker conditions for the solution of the problem. Find the only solution candidate. (You will need to distinguish between the cases a ∈ (0,1) and a ≥ 1.)

(b) The optimal value of f(x,y) will depend on a. The resulting function f*(a) is called the value function for the problem. Verify that $df(a)/da = \lambda$.