EC0138 Practice Questions for Chapter 12. "Tools for Comparative Statics" ①

Sec. 12.1.

1. Find $\frac{\partial z}{\partial t}$ by using chain rule.

(a) $F(x,y) = x + y^2$, $x = t^2$, $y = t^3$

(b) $F(x,y) = x^p y^q$, $x = at$, $y = bt$

2. Find $\frac{dz}{dt}$ when

(a) $F(x,y) = x\ln y + y\ln x$, $x = t+1$, $y = \ln t$

(b) $F(x,y) = \ln x + \ln y$, $x = Ae^{at}$, $y = Be^{bt}$

3. (a) Find $\frac{dz}{dt}$ when $z = t^2 + ye^y$ and $y = t^2$.

4. Let $Y = 10KL - \sqrt{K} - \sqrt{L}$. Suppose also that $K = 0.2t + 5$ and $L = 5e^{0.1t}$.

Find $\left(\frac{dY}{dt}\right)_{t=0}$.

Sect. 12.2.

1. Find $\frac{\partial z}{\partial t}$ and $\frac{\partial z}{\partial s}$ for $z = F(x,y) = 2x^2 + 3y^3$, $x = t^2 - s$, $y = t + 2s^3$.

2. Find $\frac{\partial z}{\partial t}$ and $\frac{\partial z}{\partial s}$ for $z = \frac{x-y}{x+y}$, $x = e^{t+s}$, $y = e^{ts}$

4. $w = x^2 + y^2 + z^2$, $x = \sqrt{t+s}$, $y = e^{ts}$, $z = s^2$

7. Demand functions of two goods are

$$\begin{cases} Q_1 = Ap_1^{-\alpha_1} p_2^{\beta_1} \\ Q_2 = Bp_1^{\alpha_2} p_2^{-\beta_2} \end{cases}$$

where p_1 & p_2 are the prices of two goods

Cost function for producing Q_1 unit of good 1 and Q_2 unit of good 2.

$$C = aQ_1 + bQ_2 + cQ_1^2$$

All constants are positive.

Find $\frac{\partial C}{\partial p_1}$ and $\frac{\partial C}{\partial p_2}$.

Sec. 12.3

1. Given $2x^2 + 6xy + y^2 = 18$. Find $\frac{dy}{dx}$.

2. Find $\frac{dy}{dx}$ for

(a) $x^2y = 1$ (b) $x - y + 3xy = 2$ (c) $y^5 - x^6 = 0$

3. A curve in the xy-plane is given by the equation $2x^2 + xy + y^2 - 8 = 0$

(a) Find $\frac{dy}{dx}$ and the equation for the tangent at the point $(2, 0)$

Sec 12.4

1. Find $\frac{\partial z}{\partial x}$ for
   (a) $3x + y - z = 0$
   (b) $xyz + xz^3 - xy^2z^5 = 1$
   (c) $e^{xyz} = 3xyz$

6. The function F is defined for all x and y by $F(x, y) = xe^{y-3} + xy^2 - 2y$. Show that the point $(1,3)$ lies on the level curve $F(x, y) = 4$, and find the equation for the tangent line to the curve at the point $(1,3)$.

Sec. 12.6

1. Show that $f(x,y) = x^4 + x^2y^2$ is homogeneous of degree 4.
2. Find the degree of homogeneity of $x(p, r) = Ap^{-1.5}r^{2.08}$
3. Show that $f(x,y) = xy^2 + x^3$ is HD3. Verify that the four properties all hold.

Sec. 12.8

1. Find the linear approximation about $(0,0)$ for
   (a) $f(x,y) = \sqrt{1+x+y}$
   (b) $f(x,y) = e^x \ln(1+y)$

4. Let $f(x,y) = 3x^2y + 2y^3$. Then $f(1,-1) = -5$. Use the approximation to estimate the value of $f(0.98, -1.01)$. How large is the error caused by this approximation?

5. (a) With $f(x,y) = 3x^2 + xy - y^2$, compute $f(1.02, 1.99)$ exactly.
   (b) Let $f(1.02, 1.99) = f(1+0.02, 2-0.01)$ and find an approximate numerical value for $f(1.02, 1.99)$. How large is the error?

Sec. 12.9

1. Determine the differential of $z = xy^2 + x^3$
2. Calculate the differentials of
   (a) $z = x^3 + y^3$, (c) $z = \ln(x^2 - y^2)$

8. Find dz when $z = Ax_1^{a_1}x_2^{a_2} \cdots x_n^{a_n}$
   where $x_1 > 0$, $x_2 > 0$, .. $x_n > 0$ and $A, a_1, a_2 \ldots a_n$ are all constant. $A > 0$.
   (Hint: First take the natural logarithm of each side)

Sec 12.10

1. Find the degrees of freedom for the following systems of equations.

(a) $xu^3 + v = y^2$
$3uv - x = 4$

(b) $x_2^2 - x_3^3 + 2y_1 - y_2^3 = 1$
$x_1^3 - x_2 + y_1^5 - y_2 = 0$

(c) $f(y+z+w) = x^3$
$x^2 + y^2 + z^2 = w^2$
$g(x, y) - z^3 = w^3$

(d) $Y = C + I + G$
$C = F(Y, T, r)$
$I = f(Y, r)$

Sec. 12.11

1. Differentiate the system

$$au + bv = cx + dy$$
$$eu + fv = gx + hy$$

with a, b, c, d, e, f, g and h are constants. $af \neq be$. Find $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$

3. Given $f_1(x_1, x_2, y_1, y_2) = 3x_1 + x_2^2 - y_1 - 3y_2^3 = 0$
$f_2(x_1, x_2, y_1, y_2) = x_1^3 - 2x_2 + 2y_1^3 - y_2 = 0$.
Find $\frac{\partial y_1}{\partial x_1}$ and $\frac{\partial y_2}{\partial x_1}$

Review

16. (a) An equilibrium model of labor demand and output pricing leads to the following system of equations

$$PF'(L) - w = 0$$
$$PF(L) - wL - B = 0$$

Here, F is twice differentiable w/ $F'(L) > 0$ and $F''(L) < 0$. All the variables are positive. Regard w and B as exogenous, so that P and L are endogenous variables, which are functions of w and B. Find expression for $\frac{\partial P}{\partial w}, \frac{\partial P}{\partial B}, \frac{\partial L}{\partial w}, \frac{\partial L}{\partial B}$ by implicit differentiation.

(b) Show that $\frac{\partial L}{\partial w} < 0$.