ECO106/138 Week 7 Lecture Note Template

Partial Elasticities

$z = f(x, y)$

$El_x z = \frac{\partial z}{\partial x} \cdot \frac{x}{z} = \frac{\partial \ln z}{\partial \ln x}$

elasticity of z w.r.t x

$El_y z = \frac{\partial z}{\partial y} \cdot \frac{y}{z} = \frac{\partial \ln z}{\partial \ln x}$

e.g.

- (Own) price elasticity of demand

$\varepsilon_p = \frac{\partial Q}{\partial P} \frac{P}{Q} = \frac{\partial \ln Q}{\partial \ln P}$

as P ↑ 1% → (ε_p)% ↓ Q

- Income Elasticity of Demand

$\varepsilon_I = \frac{\partial Q}{\partial I} \cdot \frac{I}{Q} = \frac{\partial \ln Q}{\partial \ln I}$

as I ↑ 1% → (ε_I)% ↑ D

( )% ↓ D ← if inferior good.

- Cross price elasticity of demand

$\varepsilon_c = \frac{\partial Q_A}{\partial P_B} \frac{P_B}{Q_A} = \frac{\partial \ln Q_A}{\partial \ln P_B}$ related good -

→ $\varepsilon_c > 0$ A & B Substitute. 1% P_B ↑ → Q_A ↑ by ε_c%

→ $\varepsilon_c < 0$ Q_B ↓ 1% P_B ↑ → Q_A ↓ by ε_c% complement

e.g. P_{iPod} ↑ → Q_{iPod} ↓ Q_{music} ↓ - car - gas.

e.g. Find elasticity of z with respect to x when (1) $\boxed{z = Ax^a y^b}$, (2) $z = xye^{x+y}$.

① $\varepsilon_x z = \frac{\partial z}{\partial x} \frac{x}{z}$

(1) $\varepsilon_{xz} = aAx^{a-1}y^b \cdot \frac{x}{Ax^a y^b} = a\%$

$z = Ax^a y^b$

(2) $\varepsilon_x z = \frac{ye^{x+y} + xye^{x+y}}{xye^{x+y}} \cdot x = \frac{xye^{x+y} + x^2ye^{x+y}}{xye^{x+y}} = 1 + x$

② $\varepsilon_{xz} = \frac{\partial \ln z}{\partial \ln x}$

(2) $\ln z = \ln A + a \ln x + b \ln y$

$\frac{\partial \ln z}{\partial \ln x} = a$

$e^{\ln x}$

(2) $\ln z = \ln x + \ln y + x + y$

$\frac{\partial \ln z}{\partial \ln x} = 1 + e^{\ln x} = \boxed{1+x}$ %

$A = \ln x$

$x = e^{\ln x}$

if $x = 2$ as x ↑ by 1% z ↑ by 3%

e.g. $D = Ap^{-0.28}m^{0.34}$. Find (a) price elasticity of demand and (b) income elasticity of demand. Interpret the result.

$$E_{P}D = \underbrace{-0.28}\, \cancel{Ap^{-1.28}} m^{0.34} \cdot \frac{p}{\cancel{Ap^{-0.28}} m^{0.34}} = -0.28$$

A $\quad$ B

$$\ln D = \ln A - \underbrace{0.28}\ln p + 0.34 \ln m$$

$$\frac{\partial \ln D}{\partial \ln p} = -0.28 \quad \text{i.e. } \%$$

$$\frac{\partial \ln D}{\partial p}$$

1% ↑ i.e. p → D ↓ by 0.28%

$$E_I D = \left(0.34\, A p^{-0.28} m^{-0.66}\right) \cdot \frac{m}{D}$$

$$= 0.34 > 0$$

$E_I D$ > 0 Normal good

< 0 inferior good.

$0 < E_I D < 1$ necessity

$E_I D > 1$ luxury

1% ↑ I → more than 1% ↑ in D.

income $\quad$ own price $\quad$ cross

e.g. $D_i = Am^{\alpha}p_i^{-\beta}p_j^{\gamma}$. Find (a) own price elasticity, (b) cross price elasticity and (c) income elasticity of demand. Interpret the results.

example: $\boxed{P_j}$: price of Nike shoes

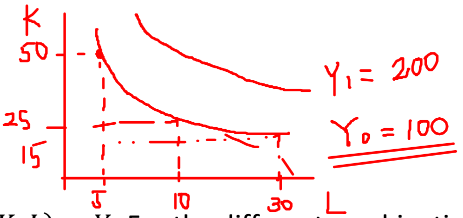
P_i: price of NB.

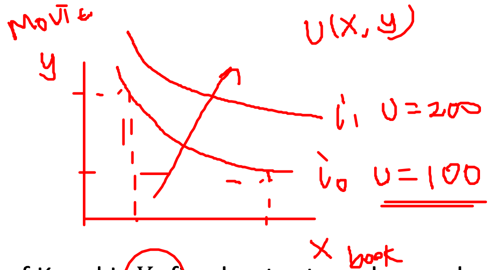
$$E_{P_j}D_i = \frac{\partial D_i}{\partial P_j}\cdot\frac{P_j}{D_i} = \frac{\partial \ln D_i}{\partial \ln P_j} =$$

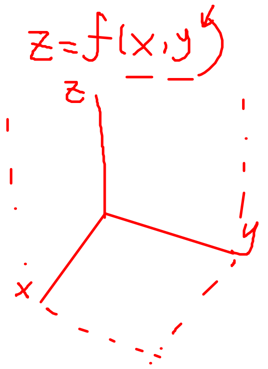
$$\ln D_i = \ln A + \alpha \ln m - \beta \ln P_i + \gamma \ln P_j$$

$$\frac{\partial \ln D_i}{\partial \ln P_j} = \gamma$$

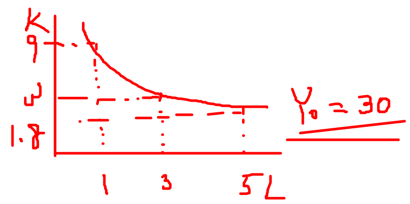
Level Curve







e.g. $F(K, L) = Y_0$ For the different combination of K and L, Y_0 fixed output can be produced.



$30 = 10K^{1/2}L^{1/2}$

$3 = K^{1/2}L^{1/2}$

$3L^{-1/2} = K^{1/2} \Rightarrow K = (3L^{-1/2})^2 = 9\frac{1}{L}$

e.g. $Y_0 = 10K^{1/2}L^{1/2}$, $\boxed{Y_0 = 30.}$

L	K	YO
1	$\boxed{9}$ capital intensive	30
2	4.5	30
3	3	30
4	2.25	30
5 Labor intensive	$\boxed{1.8}$	30

$\frac{9}{5}$

##UP TO HERE: Coverage for Midterm Exam##

Ch.12 “Tools for Comparative Statics”

1. Chain Rule

$z =$ car sales (t)
$x =$ GDP / capita (t)
$y =$ exchange rate (t)
population (t)
fuel price (t)
⋮

time series
cross section
panel data

$$z = F(x(t), y(t))$$

$$z = F(x, y) \quad \text{with } x = f(t),\ y = g(t) \text{ then}$$

$$\frac{\partial z}{\partial t} = \frac{\partial F}{\partial x}\cdot\frac{\partial x}{\partial t} + \frac{\partial F}{\partial y}\cdot\frac{\partial y}{\partial t}$$

e.g. $z = x^2 + y^3, x = t^2, y = 2t. Find\ \frac{\partial z}{\partial t}$.

$$\frac{\partial z}{\partial t} = \frac{\partial F}{\partial x}\cdot\frac{\partial x}{\partial t} + \frac{\partial F}{\partial y}\cdot\frac{\partial y}{\partial t}$$
$$= 2x(2t) + 3y^2(2)$$
$$= 2(t^2)(2t) + 3(2t)^2(2)$$
$$= 4t^3 + 24t^2$$
← as $t\uparrow$ by 1 unit, $z\uparrow$ by $(4t^3 + 24t^2)$

e.g. $z = xe^{2y}, x = \sqrt{t}, y = lnt. Find\ \frac{\partial z}{\partial t}$.

$$\frac{\partial z}{\partial t} = e^{2y}\cdot\frac{1}{2}t^{-0.5} + 2xe^{2y}\cdot\frac{1}{t}$$
$$= e^{2\ln t}\frac{1}{2}t^{-0.5} + 2\sqrt{t}\,e^{2\ln t}\frac{1}{t}$$
$$= \frac{1}{2}t^{3/2} + 2t^{3/2}$$
$$= \frac{5}{2}t^{3/2} \quad //$$

$t\uparrow$ by 1 unit → z ?
if t is given

$e^{\ln t} = t$
$(e^{\ln t})^2 = t^2$

e.g. D=D(p,m), p=p(t), m=m(t). Find the relative rate of growth of D, $\frac{\dot D}{D}$. $= \frac{\partial D}{\partial t}$

$$\dot D = \frac{\partial D}{\partial t} = \frac{\partial D}{\partial p}\cdot\frac{\partial p}{\partial t} + \frac{\partial D}{\partial m}\cdot\frac{\partial m}{\partial t}$$

$$\frac{\dot D}{D} = \underbrace{\frac{1}{D}\cdot\frac{\partial D}{\partial p}\cdot\frac{\partial p}{\partial t}}_{*\frac{p}{p}} + \underbrace{\frac{1}{D}\cdot\frac{\partial D}{\partial m}\cdot\frac{\partial m}{\partial t}}_{*\frac{m}{m}}$$

$$= \boxed{\frac{p}{D}\frac{\partial D}{\partial p}}\underbrace{\frac{\partial p}{\partial t}\cdot\frac{1}{p}}_{\frac{\dot p}{p}} + \boxed{\frac{m}{D}\frac{\partial D}{\partial m}}\cdot\underbrace{\frac{\partial m}{\partial t}\cdot\frac{1}{m}}_{\frac{\dot m}{m}}$$
⇐ What do you see here?

$$\frac{\frac{\partial D}{\partial t}}{D} \qquad \frac{\dot D}{D} = \varepsilon_{pD}\cdot\frac{\dot p}{p} + \varepsilon_{mD}\cdot\frac{\dot m}{m}$$

Chain Rule for Many Variables

time space

$$If\ z = F(x, y), with\ x = f(t, s) and\ y = g(t, s)$$

Then (a) $\frac{\partial z}{\partial t} = \frac{\partial F}{\partial x} \cdot \frac{\partial x}{\partial t} + \frac{\partial F}{\partial y} \cdot \frac{\partial y}{\partial t}$

(b) $\frac{\partial z}{\partial s} = \frac{\partial F}{\partial x} \frac{\partial x}{\partial s} + \frac{\partial F}{\partial y} \cdot \frac{\partial y}{\partial s}$

$x = x(t, s)$ $y = y(t, s)$

e.g. $z = x^2 + 2y^2, x = t - s^2, y = ts. Find\ \frac{\partial z}{\partial t}, \frac{\partial z}{\partial s}.$

$\frac{\partial z}{\partial t} = 2x(1) + 4y(s) = 2(t - s^2) + 4(ts)(s) = 2t - 2s^2 + 4st^2$

$\frac{\partial z}{\partial s} = 2x(-2s) + 4y(t) = 2(t - s^2)(-2s) + 4(ts)(t)$

$= \boxed{-4ts + 4s^3 + 4t^2 s}$

HW will be discussed in week 9

e.g. $If\ z = e^{x^2} + y^2 e^{xy}, x = 2t + 3s, y = t^2 s^3, find\ z_t'(t = 1, s = 0).$

$\frac{\partial z}{\partial t}$

General Chain Rule

e.g. $Y = F(K, L, T), K = K(t), L = L(t), T = T(t). Find\ \frac{\partial Y}{\partial t}.$

If $Y = F(K, L, T) = AK^a L^b T^c$, derive relative rate of change of output.