ECO239 Homework Answers - Probability -

Q1. a. $A \cap B = [E3, E6]$

b. $A \cup B = [E3, E4, E5, E6, E9, E10]$

c. No. It doesn't contain all of the possible sample points

Q2.

$$P = \frac{C^5_1 C^7_1}{C^{12}_2} = \frac{\frac{5!}{1!4!} \cdot \frac{7!}{1!6!}}{\frac{12!}{2!10!}} = \frac{35}{66} = 0.53$$

Q3. a. P_A = P(10% to 20% ∪ More than 20%) = 0.33 + 0.21 = 0.54

b. P_B = P(less than −10% ∪ −10% to 0%) = 0.04 + 0.14 = 0.18.

c. $P(\bar{A})$ = the rate of return will be not more than 10%.

d. $P(\bar{A}) = 1 - P_A = 0.46$

(or = 0.04 + 0.14 + 0.28 = 0.46)

e. $A \cap B = \phi$

f. $P(A \cap B) = 0$

g. $A \cup B$ = the rate of return will be less than 10%, −10% to 0%, 10% to 20% and more than 20%

h. $P(A \cup B)$ = 0.04 + 0.14 + 0.33 + 0.21 = 0.72.

(or $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ = 0.54 + 0.18 − 0 = 0.72)

i. Yes, because $P(A \cap B) = \phi$

j. No because $P(A \cup B) \neq 1$.

Q4. a. P(less than 3 defective) = $P(X<3)$ = 0.29 + 0.36 + 0.22 = 0.87.

b. P(more than 1 defective) = $P(X>1)$ = 0.22 + 0.10 + 0.03 = 0.35

Q5. $P(A) = 0.4$, $P(B) = 0.45$, $P(A \cup B) = 0.85$.

$P(A \cap B) = P(A) + P(B) - P(A \cup B) = 0.4 + 0.45 - 0.85 = 0$

($\Leftarrow$ $P(A \cup B) = P(A) + P(B) - P(A \cap B)$)

Q6. $P(A) = 0.8$, $P(B) = 0.1$, $P(A \cap B) = 0.08$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.08}{0.1} = 0.80.$$

Since $P(A|B) = P(A)$, A & B are independant.

Q7. $P(A) = 0.7$, $P(B) = 0.8$, $P(A \cap B) = 0.5$.

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.5}{0.8} = 0.625$$

Sine $P(A|B) \neq P(A)$, A & B are NOT independent.

Q8.

a. $C^5_2 = \frac{5!}{2!3!} = \frac{5 \cdot 4}{2 \cdot 1} = 10$ ← for craftman

$C^6_4 = \frac{6!}{4!\,2!} = \frac{6 \cdot 5}{2 \cdot 1} = 15$ ← for laborers.

The selections are independant, $\Rightarrow$ $10 * 15 = \underline{\underline{150}}$ possible combinatio.

b. P(select a brother who is a craftman)

$$= C^4_1 / 10 = \frac{\frac{4!}{1!3!}}{10} = \frac{4}{10}.$$

a brother is selected → out of 4 remaining craftmen select one more craftman.

P(select a brother who is a laborer)

$$= C^5_3 / 15 = \frac{5!}{3!\,2!} / 15 = \frac{5 \cdot 4}{2 \cdot 1} / 15 = \frac{10}{15}.$$

$\Rightarrow$ Multiply both probabilities

$$\frac{4}{10} * \frac{10}{15} = \frac{40}{150} = 0.2667.$$

c. P(not selecting a brother who is a craftman)

$$= 1 - \frac{4}{10} = 6/10.$$

P(not selecting a brother who is a laborer)

$$= 1 - \frac{10}{15} = \frac{5}{15}$$

$\Rightarrow$ P(~~not~~ selecting neither brother) $= 6/10 * 5/15 = \frac{30}{150} = 0.2.$

Q9. $P(A) = 0.02$, $P(B) = 0.01$, $P(C) = 0.04$
$P(B \cap C) = 0$, $P(A|B) = P(A)$, $P(A|C) = P(A)$

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C)$$
$$= 0.02 + 0.01 + 0.04 - 0.0002 - 0.0008 - 0 = 0.069.$$

$$P(A \cap B) = P(A|B) \cdot P(B) = P(A) \cdot P(B) = 0.02 * 0.01$$
$$P(A \cap C) = P(A|C) \cdot P(C) = P(A) \cdot P(C) = 0.02 * 0.04$$

Q10. A: watch tv program
B: read publication

$P(A) = 0.18$
$P(B) = 0.12$
$P(A \cap B) = 0.1$

a. $P(B|A) = \dfrac{P(A \cap B)}{P(A)} = \dfrac{0.1}{0.18} = 0.5556.$

b. $P(A|B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{0.1}{0.12} = 0.8333.$

Q11. A: problem on Monday
B: problem in the last hour of a day's shift.

$P(A) = 0.3$, $P(B) = 0.2$, $P(A \cap B) = 0.04$.

a. $P(\overline{B}|A) = \dfrac{P(A \cap \overline{B})}{P(A)} = \dfrac{P(A) - P(A \cap B)}{P(A)} = \dfrac{0.3 - 0.04}{0.3} = 0.8667.$

b. Check if $P(A \cap B) = P(A) \cdot P(B)$

Since $0.04 \neq 0.3 * 0.2 = 0.06$, the two events are not independent.

Q12.

a. $P(\text{High Inc.} \cap \text{Never}) = 0.05$

b. $P(\text{High Inc} \mid \text{Never}) = \dfrac{P(H.I \cap \text{Never})}{P(\text{Never})} = \dfrac{0.05}{0.3} = 0.1667.$

Q13. a. $P(\text{Frequent} \cap \text{Often}) = 0.12$

b. $P(\text{Frequent} \mid \text{Never}) = \dfrac{P(F \cap N)}{P(N)} = \dfrac{0.19}{(0.19 + 0.08)} = 0.7037$.

c. check if $P(N \cap F) = P(N) \cdot P(F)$

0.19

$(0.19 + 0.08) * (0.12 + 0.48 + 0.19)$

$= 0.2133$

Since $0.19 \neq 0.2133$, N & F are not independent

d. $P(\text{Often} \mid \text{Infrequent}) = \dfrac{P(O \cap I)}{P(I)} = \dfrac{0.07}{(0.07 + 0.06 + 0.08)} = 0.333$.

e. check if $P(\text{Often} \cap \text{Infrequent}) = P(\text{often}) * P(\text{Infrequent})$

$0.07 = (0.12 + 0.07) * (0.07 + 0.06 + 0.08)$

$= 0.0399$

Since $0.07 \neq 0.0399$, they are not independent.

Q14.

$P(10\% \mid Top) = 0.7$

$P(10\% \mid Middle) = 0.5$

$P(10\% \mid Bottom) = 0.2$.

a. $P(10\%) = P(10\% \cap Top) + P(10\% \cap Middle) + P(10\% \cap Bottom)$

$= P(10\% \mid Top) \cdot P(Top) + P(10\% \mid Mid) \cdot P(Mid) + P(10\% \mid BoT) \cdot P(BoT)$

$= 0.7 * \underbrace{0.25}_{\text{top quarter}} + 0.5 * \underbrace{0.5}_{\text{middle half}} + 0.2 * \underbrace{0.25}_{\text{bottom quarter}}$

$= 0.475$.

b. $P(T \mid 10\%) = \dfrac{P(10\% \cap T)}{P(10\%)} = \dfrac{P(10\% \mid T) \cdot P(T)}{P(10\%)} = \dfrac{0.7 * 0.25}{0.475}$

$= 0.3684$.

c. $P(\overline{T} \mid \overline{10\%}) = \dfrac{P(\overline{10\%} \cap \overline{T})}{P(\overline{10\%})} = \dfrac{P(\overline{10\% \cup T})}{P(\overline{10\%})} = \dfrac{1 - P(10\% \cup T)}{P(\overline{10\%})}$

$= [1 - (0.475 + 0.25 - 0.7 * 0.25)] / (1 - 0.475) = 0.857$

Q15. $P(\text{Enjoyable}) = 0.7$

$P(\text{Boring}) = 0.3$

$P(\text{strong positive} \mid \text{Enjoyable}) = 0.6$

$P(\text{strong Positive} \mid \text{Boring}) = 0.25$

a. $P(\underset{(E)}{\text{Enjoyable}} \mid \underset{(S.P)}{\text{strong positive}}) = \dfrac{P(S.P \cap E)}{P(SP)} = \dfrac{P(SP \mid E) \cdot P(E)}{P(SP)}$

$= \dfrac{0.6 * 0.7}{0.495} = 0.8485$

$P(SP) = P(SP \cap E) + P(SP \cap B)$

$= P(SP \mid E) \cdot P(E) + P(SP \mid B) \cdot P(B)$

$= 0.6 * 0.7 + 0.25 * 0.3 = 0.495$

$P(3E \mid SP) = [P(E \mid SP)]^3 = (0.8485)^3 = 0.6109$

↑ take 3 classes

b. $P(\text{at least } 1E \mid S.P) = 1 - P(\text{no } E \mid S.P)$

$= 1 - [\underbrace{P(B \mid SP)}]^3 = 1 - (0.1515)^3 = 0.9965$

$\parallel$

$\dfrac{P(SP \cap B)}{P(SP)} = \dfrac{P(SP \mid B) \cdot P(B)}{P(SP)}$

$= \dfrac{0.25 * 0.3}{0.495} = 0.1515$

Q16. $P(A_1) = 0.4$, $P(B_1 \mid A_1) = 0.6$

$P(B_1 \mid A_2) = 0.7$.

$P(A_1 \mid B_1) = \dfrac{P(B_1 \cap A_1)}{P(B_1)} = \dfrac{P(B_1 \mid A_1) \cdot P(A_1)}{P(B_1)} = \dfrac{0.6 * 0.4}{0.66} = 0.3636$

$\parallel$

$P(B_1 \cap A_1) + P(B_1 \cap A_2)$

$= P(B_1 \mid A_1) \cdot P(A_1) + P(B_1 \mid A_2) \cdot P(A_2)$

$= 0.6 * 0.4 + 0.7 * 0.6$

$= 0.66$

Q17.

$P(A_1) = 0.6$, $P(B1 \mid A1) = 0.6$, $P(B1 \mid A2) = 0.4$

$$P(A_1 \mid B_1) = \frac{P(A_1 \cap B_1)}{P(B_1)} = \frac{P(B_1 \mid A_1) \cdot P(A_1)}{P(B_1 \cap A_1) + P(B_1 \cap A_2)}$$

$$= \frac{0.6 * 0.6}{P(B_1 \mid A_1) \cdot P(A_1) + P(B_1 \mid A_2) P(A_2)}$$

$$= \frac{0.36}{0.36 + 0.4 * 0.4} = 0.6923$$

Q18.

E_1: Stock performs much better than market average

$E2$: Stock performs as same as the average

E_3: stock performs worse than the market average

A: Stock is rated a "Buy".

$P(E_1) = 0.25$, $P(E2) = 0.5$, $P(E_3) = 0.25$.

$P(A \mid E_1) = 0.4$, $P(A \mid E_2) = 0.2$, $P(A \mid E_3) = 0.1$.

$$P(E_1 \mid A) = \frac{P(E_1 \cap A)}{P(A)} = \frac{P(A \mid E_1) \cdot P(E_1)}{P(A \cap E_1) + P(A \cap E_2) + P(A \cap E_3)}$$

$$= \frac{P(A \mid E_1) \cdot P(E_1)}{P(A \mid E_1) \cdot P(E_1) + P(A \mid E_2) \cdot P(E_2) + P(A \mid E_3) \cdot P(E_3)}$$

$$= \frac{0.4 * 0.25}{0.4 * 0.25 + 0.2 * 0.5 + 0.1 * 0.25}$$

$$= 0.444$$