ECO 239 HW Answers 2. [Describing Data: Numerical]

1. a. $$\frac{21+22+27+36+18+19+22+23+22+28+36+33}{12}$$

$= 23.75$

b. step1: re-order:

18 19 21 22 22 22 23 27 28 33 36 36

step2: Median Position

$$M.P = \frac{12+1}{2} = 6.5$$

step3. Median.

$$\frac{22+23}{2} = 22.5$$

c. Mode = 22

d. Since Mean > Median, right (positively) skewed.)

2. $$S^2 = \frac{\Sigma (X_i - \bar{X})^2}{n-1}, \quad S = \sqrt{\frac{\Sigma (X_i - \bar{X})^2}{n-1}} = \sqrt{S^2}$$

step1. Compute $\bar{X} = \frac{6+8+7+10+3+5+9+8}{8} = 7$

step2. Compute $(X_i - \bar{X})$.

step3. Compute $(X_i - \bar{X})^2$.

step4. Sum $(X_i - \bar{X})^2$

step4. devide step4 by $n-1 = 7$

ID	Data	$X_i - \bar{X}$	$(X_i - \bar{X})^2$
1	6	-1	1
2	8	1	1
3	7	0	0
4	10	3	9
5	3	-4	16
6	5	-2	4
7	9	2	4
8	8	1	1
			36

$$S^2 = \frac{36}{7} = 5.143$$

$$S = 2.268$$

$n = 22$

3. a. Location of Q1 $(LQ1) = 0.25(22+1) = 5.75$.

Location of Q3 $(LQ3) = 0.75(23) = 17.25$.

$Q1 = 48 + (50-48) * 0.75 = 49.5$

(between 48 and 50)

$Q3 = 73 + (76-73) * 0.25 = 73.75$.

(between 73 and 76)

$IQR = Q3 - Q1 = 73.75 - 49.5 = 24.25$.

b. Five number summary

Min	Q1	Median	Q3	Max
30	49.5	58.5	73.75	87

(L.Q.2 = $0.5(23) = 11.5$. (between 58 and 59)

Median = Q2 = $58 + (59-58) * 0.5 = 58.5$.)

c. 92nd percentile

Location = $0.92(23) = 21.16$

(between 83 and 87)

92nd Per. = $83 + (87-83) * 0.16 = 83.64$.

4. $\mu = 250$, $\sigma = 20$ $\quad \mu \pm k\sigma \Rightarrow 250 \pm 20k$.

For a, [190 310]

$250 - 20k = 190$, $250 + 20k = 310$.

$\boxed{k = 3}$ → at least $100\left[1 - \frac{1}{k^2}\right]\%$

$= 100\left(1 - \frac{1}{9}\right)\%$

$= 89\%$.

4 (contd)

For b, $[210 \cdot 290]$

$250 - 20k = 210$, $250 + 20k = 290$

$k = 2 \rightarrow$ at least $100\%\left[1 - \frac{1}{k^2}\right]$

$= 100 * \left(1 - \frac{1}{4}\right)$

$= \underline{\underline{75\%}}$

For C, $[230 \cdot 270]$

$230 - 20k = 230$, $250 + 20k = 270$

$k = 1 \rightarrow$ at least $100\%\left[1 - \frac{1}{1}\right]$

$= 0\%$.

(← Chevychev's theorem doesn't work for $k=1$.)

5. a. $\frac{4.6 + 3.2 + 5.4 + 2.6 + 5.2}{5} = 4.2$.

b. $\frac{4.6 * 8 + 3.2 * 3 + 5.4 * 6 + 2.6 * 2 + 5.2 * 5}{8 + 3 + 6 + 2 + 5}$

$= 4.583$.

6.

X	Y	$x_i - \bar{X}$	$(x_i - \bar{X})^2$	$y_i - \bar{y}$	$(y_i - \bar{y})^2$	$(x_i - \bar{X})(y_i - \bar{y})$
12	200	-7	49	-156	24336	1092
30	600	11	121	244	59536	2684
15	270	-4	16	-86	7396	344
24	500	5	25	144	20736	720
14	210.	-5	25	-146	21316	730
$\bar{X} = 19$	$\bar{y} = 356$		236		133320	5570

$Cov(X,Y) = S_{xy} = \frac{5570}{4} = 1392.5$

$r_{xy} = \frac{cov(X,Y)}{S_x S_y} = \frac{1392.5}{\sqrt{\frac{236}{4}}\sqrt{\frac{133320}{4}}} = 0.993$.

7.

X	Y	$(X_i-\bar{X})$	$(Y_i-\bar{Y})$	$(X_i-\bar{X})^2$	$(Y_i-\bar{Y})^2$	$(X_i-\bar{X})(Y_i-\bar{Y})$
6	80	-2	30	4	900	-60
7	60	-1	10	1	100	-10
8	70	0	20	0	400	0
9	40	1	-10	1	100	-10
10	0	2	-50	4	2500	-100
				10	4000	-180

$\bar{X} = 8 \quad \bar{Y} = 50$

$$Cov(X,Y) = S_{XY} = \frac{-180}{4} = -45.$$

$$r_{XY} = \frac{Cov(X,Y)}{S_X S_Y} = \frac{-45}{\sqrt{\frac{10}{4}}\sqrt{\frac{4000}{4}}} = -0.9.$$

8. $b_1 = \frac{Cov(X,Y)}{S_X^2}$, $b_0 = \bar{y} - b_1\bar{X}$.

X	Y	$(X_i-\bar{X})$	$(Y_i-\bar{Y})$	$(X_i-\bar{X})^2$	$(X_i-\bar{X})(Y_i-\bar{Y})$
1	5	-3	-2	9	6
3	7	-1	0	1	0
4	6	0	-1	0	0
5	8	1	1	1	1
7	9	3	2	9	6
				20	13

$\bar{X} = 4 \quad \bar{Y} = 7$

$$Cov(X,Y) = S_{XY} = \frac{13}{4} = 3.25.$$

$$S_X^2 = Var(X) = \frac{20}{4} = 5$$

a. $b_1 = \frac{3.25}{5} = 0.65$

b. $b_0 = \bar{y} - 0.65\bar{X} = 7 - 0.65 * 4$

$= 4.4$

c. $Y = 4.4 + 0.65X$

9. (5)

X	Y	$(X_i-\bar{X})$	$(Y_i-\bar{Y})$	$(X_i-\bar{X})^2$	$(Y_i-\bar{Y})^2$	$(X_i-X)(Y_i-Y)$
10	100	2.86	-38.57	8.18	1487.64	-110.31
8	120	0.86	-18.57	0.74	344.84	-15.97
5	200	-2.14	61.43	4.58	3773.64	-131.46
4	200	-3.14	61.43	9.86	3773.64	-192.89
10	90	2.86	-48.57	8.18	2359.04	-138.91
7	110	-0.14	-28.57	0.02	816.24	4.00
6	150	-1.14	11.43	1.30	130.64	-13.03
				32.86	12685.68	-598.57

$\bar{X}=7.14$ $\bar{Y}=138.57$

a. $Cov(X,Y) = S_{XY} = \frac{-598.57}{6} = -99.76$

$r_{XY} = \frac{Cov(X,Y)}{S_X S_Y} = \frac{-99.76}{\sqrt{\frac{32.86}{6}}\sqrt{\frac{12685.68}{6}}} = -0.927$

b. $b1 = \frac{Cov(X,Y)}{S_X^2} = \frac{-99.76}{\left(\frac{32.86}{6}\right)} = -18.22$

c. $b0 = \bar{Y} - b1\bar{X} = 138.57 - (-18.22) * 7.14$

$= 268.66$

d. $Y = 268.66 - 18.22X$

if $X = 7$, $Y = 141.12$