ECO 239 HW 4 "Discrete Random Variables and Probability Distributions" part 1.

<u>Answers</u>

①

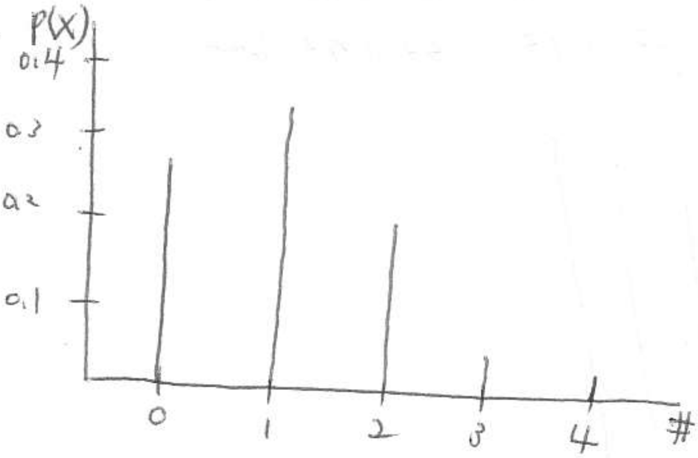
Q1. a)

X	0	1	2	3	4	5	6	7	8	9
P(X)	0.10	0.08	0.07	0.15	0.12	0.08	0.10	0.12	0.08	0.10
→ F(X)	0.10	0.18	0.25	0.40	0.52	0.60	0.70	0.82	0.90	1.00

b) $P(X \geq 5) = 0.08 + 0.10 + 0.12 + 0.08 + 0.10 = 0.48$

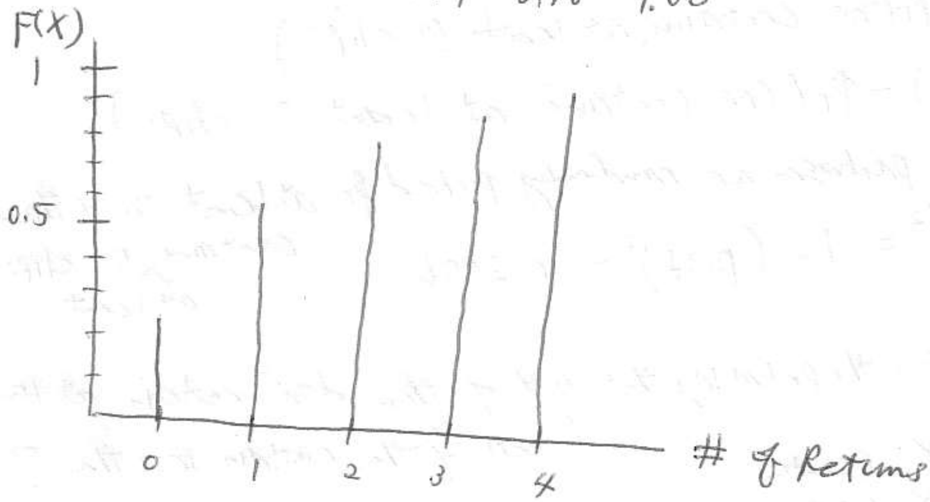
c) $P(3 \leq X \leq 7) = 0.15 + 0.12 + 0.08 + 0.10 + 0.12 = 0.57$.

Q2. a.



b.

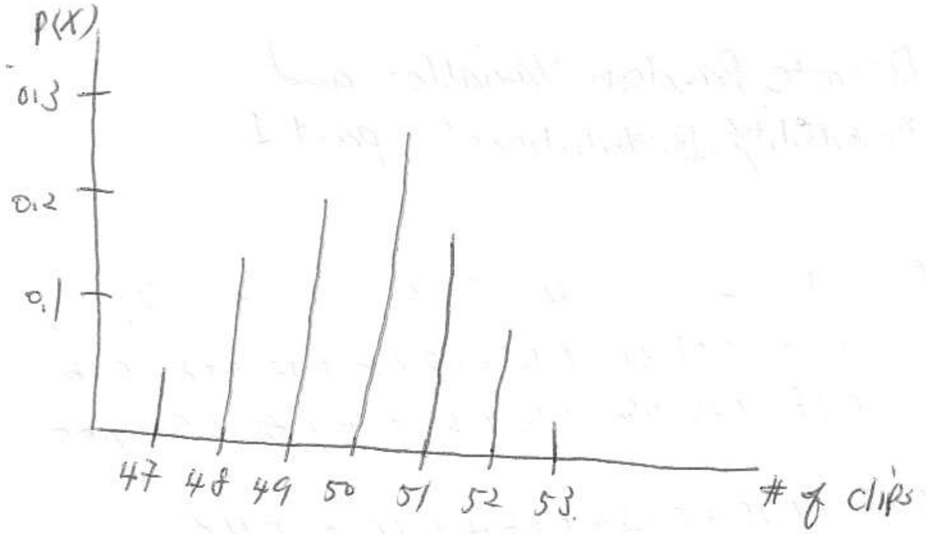
# Returns	0	1	2	3	4
P(X)	0.28	0.36	0.23	0.09	0.04
F(X)	0.28	0.64	0.87	0.96	1.00



c. $\mu = 0*0.28 + 1*0.36 + 2*0.23 + 3*0.09 + 4*0.04$
$= 1.25$ defects

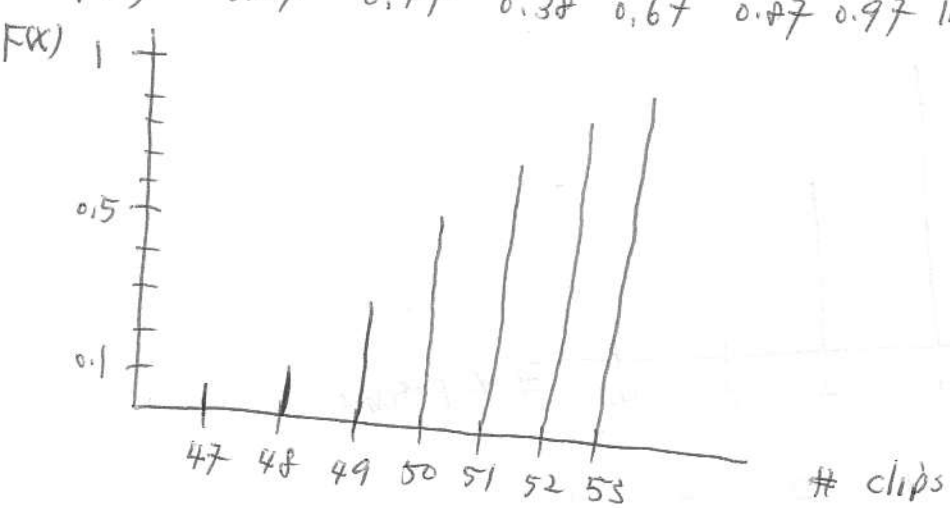
d. $\sigma^2 = \Sigma x^2 P(x) - \mu_x^2 = 0^2 \cdot 0.28 + 1^2 \cdot 0.36 + 2^2 \cdot 0.23 + 3^2 \cdot 0.09 + 4^2 \cdot 0.04 - (1.25)^2$
$= 1.1675$.

Q3. a.



b.

# clips	47	48	49	50	51	52	53
P(X)	0.04	0.13	0.21	0.29	0.20	0.10	0.03
F(X)	0.04	0.17	0.38	0.67	0.87	0.97	1.00



c. $P(49 \le X \le 51) = 0.21 + 0.29 + 0.20 = 0.70$.

d. $P(X \geq 50) = Pr$(one contains at least 50 clips)

$1 - P(X<50) = Pr$(one contains at least 50 clips)

Since now two packages are randomly picked Pr(at least one of them contains at least 50 clips) ⇒

$1 - [\underbrace{P(X<50)}_{0.38}]^2 = 1 - (0.38)^2 = 0.8556$.

This is the probability that both of them dont contain at least 50. (both of them or contain less than 50 clips)

e.

$C = 16 + 2X$ (cents)

$R = 150$ (cents)

$\pi = R - C = 150 - (16+2X) = 134 - 2X$ (cents)

$\boxed{\mu_\pi = 134 - 2\mu_X = 134 - 2(49.9) = 34.2}$

($\mu_X = 0.04 \cdot 47 + 0.13 \cdot 48 + 0.21 \cdot 49 + 0.29 \cdot 50 + 0.20 \cdot 51 + 0.10 \cdot 52 + 0.03 \cdot 53$
$= 49.9$

$\sigma_\pi^2 = 4\sigma_X^2 = 4(1.95) = 7.8$ $\boxed{\sigma_\pi = \sqrt{7.8} = 2.79}$

$\sigma_X^2 = 0.04(47)^2 + 0.13(48)^2 + 0.21(49)^2 + 0.29(50)^2 + 0.20(51)^2 + 0.10(52)^2 + 0.03(53)^2 - (49.9)^2$

③

Q4 a. X ← # of defectives from 2 picks.

X	0	1	2
$P(X)$	0.81	0.18	0.01

Pr(defective) = 0.1.
⇒ Pr(Non-defective) = 0.9

$P(0) = P(\text{Non defect}) \cdot P(\text{Non defect}) = 0.9 * 0.9 = 0.81$

$P(1) = P(\text{Non def.}) \cdot P(\text{Def.}) + P(\text{Def.}) \cdot P(\text{Non def.}) = 0.9 * 0.1 + 0.1 * 0.9 = 0.18$

$P(2) = P(\text{def}) \cdot P(\text{def}) = 0.1 * 0.1 = 0.01$

b.

Y	0	1	2
$P(Y)$	0.805	0.189	0.005

$P(Y=0) = \frac{18}{20} * \frac{17}{19} = \frac{153}{190} = 0.805$

$P(Y=1) = \left(\frac{2}{20} * \frac{18}{19}\right) + \left(\frac{18}{20} * \frac{2}{19}\right) = \frac{36}{190} = 0.189$

$P(Y=2) = \left(\frac{2}{20} * \frac{1}{19}\right) = \frac{1}{190} = 0.005$

c. $\mu_X = 0 \cdot 0.81 + 1 \cdot 0.18 + 2 \cdot 0.01 = 0.18 + 0.02 = 0.2$

$\sigma_X^2 = 0^2 \cdot 0.81 + 1^2 \cdot 0.18 + 4 \cdot 0.01 - (0.2)^2 = 0.18$

d. $\mu_Y = 0 \cdot 0.805 + 1 \cdot 0.189 + 2 \cdot 0.005 = 0.199 \ (\approx 0.2)$

$\sigma_Y^2 = 0^2 \cdot 0.805 + 1^2 \cdot 0.189 + 2^2 \cdot 0.005 - (0.199)^2 = 0.1694$

(or $0 \cdot \frac{153}{190} + 1 \cdot \frac{36}{190} + 4 \cdot \frac{1}{190} - (0.2)^2 = 0.1705$)

Q5. a. $\mu_X = 0 \cdot 0.1 + 1 \cdot 0.26 + 2 \cdot 0.42 + 3 \cdot 0.16 + 4 \cdot 0.06 = 1.82$

$\sigma_X^2 = 0.26 + 4 \cdot 0.42 + 9 \cdot 0.16 + 16 \cdot 0.06 - (1.82)^2 = 1.0276$

$\sigma_X = \sqrt{1.0276} = 1.0137$

b. $C = 1500 X$

$\mu_C = 1500 * \mu_X = 1500 * 1.82 = 2730$

$\sigma_C = 1500 \sigma_X = 1520.55$

Q6. $\mu_X = E(X) = \Sigma x P(x) = 0 \cdot 0.5 + 1 \cdot 0.5 = 0.5$

$\sigma_X^2 = p(1-p) = 0.5 \cdot 0.5 = 0.25$.

Q7. $P(X=7) = \frac{14!}{7!\,7!}(0.3)^7 (0.7)^7 = \frac{14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8}{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2}(0.3)^7 (0.7)^7$

$p = 0.3$
$n = 14$

$= 3432 \cdot (0.0002187)(0.08235)$
$= 0.06181$.

$P(X<6) = P(X \le 5) = P(X=0) + P(X=1) + P(X=2) + \cdots + P(X=5)$

$= \frac{14!}{0!\,14!}(0.3)^0 (0.7)^{14} + \frac{14!}{1!\,13!}(0.3)^1 (0.7)^{13} + \frac{14!}{2!\,12!}(0.3)^2 (0.7)^{12}$
$+ \frac{14!}{3!\,11!}(0.3)^3 (0.7)^{11} + \frac{14!}{4!\,10!}(0.3)^4 (0.7)^{10} + \frac{14!}{5!\,9!}(0.3)^5 (0.7)^9$.

$= 0.7805$.

[* Sorry for including this calculation. I won't ask you to compute this much stuff in the exam. But, be familiar w/ the Binomial formula.]

Q7. a. $p = 0.25$, $n = 5$.

$P(X \ge 1) = 1 - P(X<1) = 1 - P(X=0)$
$= 1 - \frac{5!}{0!\,5!}(0.25)^0 (0.75)^5 = 1 - (0.75)^5 = 0.7627$.

b. $P(X \ge 3) = 1 - P(X \le 2) = 1 - (P(X=0) + P(X=1) + P(X=2))$

$= 1 - \left[\frac{5!}{0!\,5!}(0.25)^0 (0.75)^5 + \frac{5!}{1!\,4!}(0.25)^1 (0.75)^4 + \frac{5!}{2!\,3!}(0.25)^2 (0.75)^3\right]$

$= 1 - 0.8965 = 0.1035$.

Q9. $p = 0.4$. $n = 5$

a. $P(X=5) = \frac{5!}{5!\,0!}(0.4)^5 \cdot (0.6)^0 = 0.1024$.

b. $P(X \geq 3) = 1 - P(X \leq 2) = 1 - (P(X=0) + P(X=1) + P(X=2))$

$= 1 - \left(\frac{5!}{0!\,5!}(0.4)^0(0.6)^5 + \frac{5!}{1!\,4!}(0.4)^1(0.6)^4 + \frac{5!}{2!\,3!}(0.4)^2(0.6)^3\right)$

$= 1 - 0.6826 = 0.3174$.

c. $P(X \geq 2) = 1 - P(X \leq 1) = 1 - (P(X=0) + P(X=1))$

* $\boxed{n=4}$

$= 1 - \left(\frac{4!}{0!\,4!}(0.4)^0(0.6)^4 + \frac{4!}{1!\,3!}(0.4)^1(0.6)^3\right)$

$= 1 - 0.4752 = 0.5248$.

d. $E(X) = np = 5 * 0.4 = 2$ games

e. $E(X) = 1 + \underbrace{np}_{\substack{n=4 \\ p=0.4}} = 1 + 4 * 0.4 = 2.6$ games.

Q10. $p = 0.78$, $n = 620$

a. $E(X) = np = 620 * 0.78 = 483.6$.

$\sigma^2(X) = np(1-p) = 620(0.78)(1-0.78) = 106.392$

$\sigma(X) = \sqrt{106.392} = 10.3146$.

b. $C = 2X$

$E(C) = 2(483.6) = 967.2$

$\sigma_C = 2(10.3146) = 20.6292$.

Q11. $p = 0.2$.

Rule 1: $P(X=0) = \frac{10!}{0!\,10!}(0.2)^0(0.8)^{10} = 0.1074$ $(n=10)$

Rule 2: $P(X \leq 1) = P(X=0) + P(X=1)$ $(n=20)$

$= \frac{20!}{0!\,20!}(0.2)^0(0.8)^{20} + \frac{20!}{1!\,19!}(0.2)^1(0.8)^{19} = 0.0692$.

$\Rightarrow$ Rule2 has the smaller [illegible]