ECO239 HW5 Answers "Discrete R.V. & Probability Distribution" part 2 ①

Q1 a.

Y (identify)

		0	1	P(X)
X (regularly)	0	$P(0,0) = 0.84-0.3 = 0.54$	$P(0,1) = 0.45-0.15 = 0.3$	$P(X=0) = 1-0.16 = 0.84$
	1	$P(1,0) = 0.55-0.54 = 0.01$	ⓐ $P(1,1) = 0.15$	ⓑ $P(X=1) = 0.16$
P(Y)		$P(Y=0) = 1-0.45 = 0.55$	ⓒ $P(Y=1) = 0.45$	1

ⓐ, ⓑ, ⓒ are given in the question.

* joint probability function is found as $P(0,0)$, $P(0,1)$, $P(1,0)$, $P(1,1)$.
$P(X=0)$, $P(X=1)$, $P(Y=0)$, $P(Y=1)$ are marginal probability

b. $P(Y=0 \mid X=1) = \dfrac{P(1,0)}{P(X=1)} = \dfrac{0.01}{0.16} = 0.0625$.

$P(Y=1 \mid X=1) = \dfrac{P(1,1)}{P(X=1)} = \dfrac{0.15}{0.16} = 0.9375$.

c. $E[XY] = \Sigma XY\, P(x,y)$

$= 0\cdot 0\cdot 0.54 + 0\cdot 1\cdot 0.3 + 1\cdot 0\cdot 0.01 + 1\cdot 1\cdot 0.15 = 0.15$.

$\mu_X = \Sigma X P(X) = 0\cdot 0.84 + 1\cdot 0.16 = 0.16$

$\mu_Y = \Sigma Y P(Y) = 0\cdot 0.55 + 1\cdot 0.45 = 0.45$

$Cov(X,Y) = 0.15 - 0.16 * 0.45 = \boxed{0.078}$.

There is a positive relationship between brand watchers of a late-night talk show and brand name recognition.

Q2. $P(X \geq 4) = 1 - P(X \leq 3) = 1 - \underbrace{0.5} = 0.5$

$n=7,\ p=0.5$ → from Cumulative Binomial Table.

Q3. Step 1. Find the probability of overbooking a flight.
Define p as the probability of a ticketed passenger showing up for a flight. $p = 1-0.2 = 0.8$. → Use Binomial Table for $n=10$, $p=0.8$. Becareful!
Probability
Since 10% of the time 9 tickets are sold and 5% of the time 10 tickets are sold, the proportion of flights where the number of ticketed passengers showing up exceeds the number of available seats ($=8$) is

$0.1 * P(X=9) + 0.05 * P(X=10)$ Convert this probability as $P(X=n-X \mid n=10, p=1-0.8) + 0$

$= 0.1 * \underbrace{P(X=9 \mid n=10, p=0.8)} + 0.05 * P(X=10 \mid n=10, p=0.8)$ use table for $p=0.2$

$= 0.1 * P(X=1 \mid n=10, p=0.2) + 0.05 * P(X=0 \mid n=10, p=0.2)$

$= 0.1 * 0.2684 + 0.05 * 0.1074 = 0.03221$.

Q4 $N=80,\ n=20,\ S=42,\ X=9$

$$P(X=9) = \frac{C^{42}_{9}\ C^{38}_{11}}{C^{80}_{20}} = \frac{\frac{42!}{9!\,33!}\ \frac{38!}{11!\,27!}}{\frac{80!}{20!\,60!}} =$$

$$= \frac{445891810 * 1203322288}{\frac{80!}{20!\,60!}} = 0.151769$$

Q5. $N=16,\ S=8,\ n=8,\ X=4$

$$P(X=4) = \frac{C^8_4 C^8_4}{C^{16}_8} = \frac{\frac{8!}{4!\,4!}\ \frac{8!}{4!\,4!}}{\frac{16!}{8!\,8!}} = \frac{70 \times 70}{12870} = 0.38073$$

Q6. $N=10,\ n=6,\ S=5$

$$P(X<3) = P(X\le 2) = \cancel{\underbrace{\frac{C^5_0\, C^5_6}{C^{10}_6}}_{P(X=0)}} + \underbrace{\frac{C^5_1\, C^5_5}{C^{10}_6}}_{P(X=1)} + \underbrace{\frac{C^5_2\, C^5_4}{C^{10}_6}}_{P(X=2)}$$

true for discrete R.V.

$$= \frac{5}{210} + \frac{50}{210} = \frac{55}{210} = 0.2619$$

Q7. $\lambda = 4.2$

$$P(X\ge 3) = 1 - P(X\le 2) = 1 - [P(X=0) + P(X=1) + P(X=2)]$$

$$= 1 - \left[\frac{e^{-4.2}(4.2)^0}{0!} + \frac{e^{-4.2}(4.2)^1}{1!} + \frac{e^{-4.2}(4.2)^2}{2!}\right]$$

$$= 1 - \left[e^{-4.2}\left(1 + 4.2 + \frac{(4.2)^2}{2}\right)\right]$$

$$= 1 - 0.2102 = 0.7898$$

Q8. $\lambda = 0.055 * 100 = 5.5$. ← Use Poisson Approximation

$$P(X<3) = P(X\le 2) = \frac{e^{-5.5}(5.5)^0}{0!} + \frac{e^{-5.5}(5.5)^1}{1!} + \frac{e^{-5.5}(5.5)^2}{2!}$$

$$= e^{-5.5}\left(1 + 5.5 + \frac{(5.5)^2}{2}\right)$$

$$= 0.088376$$

Q9. HW5 3

a)

X (Friday)

Y (away)	0	1	P(Y)
0	⑤ 0.704	③ 0.168	0.872
1	④ 0.096	② 0.032	0.128
P(X)	0.80	① 0.20	1

① From the question "one-fifth of which are on Fridays".

② From the question, we know

$P(Y=1 \mid X=1) = 0.16.$

$\Rightarrow P(X=1, Y=1) = 0.16 * P(X=1)$

$= 0.16 * 0.2$ ①

$= 0.032$

③ ① − ② = 0.2 − 0.032 = 0.168.

④ From the question, we know

$P(Y=1 \mid X=0) = 0.12$

$\Rightarrow P(X=0, Y=1) = 0.12 * P(X=0)$

$= 0.12 * 0.8 \leftarrow 1 - 0.20$ ①

$= 0.096.$

⑤ 0.80 − 0.096 = 0.704.

b.

$P(Y \mid X=0)$

$P(Y=0 \mid X=0) = \frac{P(0,0)}{P(X=0)} = \frac{0.704}{0.8} = 0.88$

$P(Y=1 \mid X=0) = 0.12 \leftarrow$ given in the question

c. Marginal probability functions are given in the table above.

d.

$Cov(X,Y) = E[XY] - \mu_X \mu_Y$

$E[XY] = \sum xy\, P(x,y) = 0 \cdot 0 \cdot 0.704 + 0 \cdot 1 \cdot 0.168 + 0 \cdot 1 \cdot 0.096 + 1 \cdot 1 \cdot 0.032$

$= \underline{0.032}$

$\mu_X = 0 \cdot 0.8 + 1 \cdot 0.2 = \underline{0.2}$

$\mu_Y = 0 \cdot 0.872 + 1 \cdot 0.128 = \underline{0.128}.$

$Cov(X,Y) = 0.032 - 0.2 * 0.128 = 0.0064$ //

There is a positive relationship between X and Y, professors are more likely to be away from the office on Friday than during the other days.

Q10. $\mu = 5\mu_X + 10\mu_Y = 5(2.38) + 10(1.65) = \underline{28.4}$

$$\begin{cases} \mu_X = 0\cdot 0.08 + 1\cdot 0.16 + 2\cdot 0.28 + 3\cdot 0.32 + 4\cdot 0.10 + 5\cdot 0.06 = 2.38 \\ \mu_Y = 0\cdot 0.18 + 1\cdot 0.26 + 2\cdot 0.36 + 3\cdot 0.13 + 4\cdot 0.07 = 1.65 \end{cases}$$

$\sigma^2 = \underbrace{(5)^2\sigma_X^2 + (10^2)\sigma_Y^2}_{\leftarrow \text{Independent} \rightarrow Cov(X,Y)=0} = 25(1.5965) + 100(1.2675) = 166.6625$

$\sigma_X^2 = \Sigma X^2 P(X) - \mu_X^2 = 0^2\cdot 0.08 + 1^2\cdot 0.16 + 2^2\cdot 0.28 + 3^2\cdot 0.32 + 4^2\cdot 0.10 + 5^2\cdot 0.06 - (2.38)^2 = 1.5965$

$\sigma_Y^2 = \Sigma Y^2 P(Y) - \mu_Y^2 = 0^2\cdot 0.18 + 1^2\cdot 0.26 + 2^2\cdot 0.36 + 3^2\cdot 0.13 + 4^2\cdot 0.07 - (1.65)^2 = 1.2675$

$\sigma = \sqrt{166.6625} = \underline{\underline{12.9098}}$

Q11. a. $P(X=3 \mid n=5, p=0.55) = \frac{5!}{3!\,2!}(0.55)^3(0.45)^2 = 0.3369$

$\leftarrow$ Binomial Distribution

b. $P(X \geq 3 \mid n=5, p=0.55) = P(X=3) + P(X=4) + P(X=5)$

$= \frac{5!}{3!\,2!}(0.55)^3(0.45)^2 + \frac{5!}{4!\,1!}(0.55)^4(0.45)^1 + \frac{5!}{5!\,0!}(0.55)^5(0.45)^0$

$= 0.3369 + 0.2059 + 0.0503$

$= 0.5931$

c. $\mu = np = 80(0.55) = 44$. The proportion is $44/80 = 0.55$.

$\sigma = \sqrt{np(1-p)} = \sqrt{80(0.55)(0.45)} = 4.4497$.

The proportion is $4.4497/8 = 0.5562$ //

Q12.

$W = 10P_X - 5P_Y$

$\mu_W = 10\mu_{P_X} - 5\mu_{P_Y}$

$= 10(4) - 5(6) = \underline{\underline{10}}$

$\mu_{P_X} = 3(0.25) + 4(0.5) + 5(0.25) = 4$

$\mu_{P_Y} = 4(0.3) + 6(0.4) + 8(0.3) = 6$

Marginal probabilities

$\sigma_W^2 = (10)^2\sigma_{P_X}^2 + (-5)^2\sigma_{P_Y}^2 + 2(10)(-5)Cov(P_X, P_Y)$

$= 100\sigma_{P_X}^2 + 25\sigma_{P_Y}^2 - 100\,Cov(P_X, P_Y)$

$= 100(0.5) + 25(2.4) - 100(0.2)$

$= \underline{\underline{90}}$

$\sigma_{P_X}^2 = \Sigma P_X^2 P(P_X) - (\mu_{P_X})^2$

$= 9(0.25) + 16(0.5) + 25(0.25) - 16$

$= 0.5$

$\sigma_{P_Y}^2 = 16(0.3) + 36(0.4) + 64(0.3) - 36 = 2.4$

$Cov(P_X, P_Y) = \Sigma\Sigma P_X P_Y P(P_X P_Y) - \mu_X\mu_Y$

$= 3\cdot 4\cdot 0.1 + 3\cdot 6\cdot 0.1 + 3\cdot 8\cdot 0.05$

$+ 4\cdot 4\cdot 0.15 + 4\cdot 6\cdot 0.2 + 4\cdot 8\cdot 0.15$

$+ 5\cdot 4\cdot 0.05 + 5\cdot 6\cdot 0.1 + 5\cdot 8\cdot 0.1$

$- 4 * 6 = 0.2$