ECO239   HW 6 "Continuous Random Variables and Probability Distributions" Part II

Q1. Let the random variable Z follow a standard normal distribution.
   a. The probability is 0.80 that Z is less than what number?
   b. The probability is 0.30 that Z is less than what number?
   c. The probability is 0.2 that Z is greater than what number?
   d. The probability is 0.6 that Z is greater than what number?

Q2. Let the random variable X follow a normal distribution with mean 80 and variance 100.
   a. Find the probability that X is greater than 55.
   b. Find the probability that X is greater than 68 and less than 85.
   c. Find the probability that X is less than 67.
   d. The probability is 0.1 that X is greater than what number?
   e. The probability is 0.08 that X is in the symmetric interval about the mean between which two numbers?

Q3. It is known that amounts of money spent on textbooks in a year by students on a particular campus follow a normal distribution with mean $380 and standard deviation $50.
   a. What is the probability that a randomly chosen student will spend less than $400 on textbooks in a year?
   b. What is the probability that a randomly chosen student will spend more than $360 on textbooks in a year?
   c. Draw a graph to illustrate why the answers to parts (a) and (b) are the same.
   d. What is the probability that a randomly chosen student will spend between $300 and $400 on textbooks in a year?
   e. You want to find a range of dollar spending on textbooks in a year that includes 80% of all students on this campus. Explain why any number of such ranges could be found, and find the shortest one.

Q4. The tread life of a particular brand of tire has a normal distribution with mean 35,000 miles and standard deviation 4,000 miles.
   a. What proportion of these tires has a tread life of more than 38,000 miles?
   b. What proportion of these tires has a tread life of less than 32,000 miles?
   c. What proportion of these tires has a tread life of between 32,000 and 38,000 miles?
   d. Draw a graph of the probability density function of tread lives, illustrating:
      i. Why the answers to parts (a) and (b) are the same.
      ii. Why the answers to parts (a), (b) and (c) sum to 1.

Q5. Scores on a test follow a normal distribution. What is the probability that a randomly selected student will achieve a score that exceeds the mean score by more than 1.5 standard deviations?

Q6. I am considering two alternative investments. In both cases I am unsure about the percentage return but believe that my uncertainty can be represented by normal distributions with the means and standard deviations shown in the accompanying table. I want to make the investment that is more likely to produce a return of at least 10%. Which should I choose?

	Mean	Standard Deviation
Investment A	10.4	1.2
Investment B	11.0	4.0

Q7. Given a random sample of size n = 4,000 from a binomial probability distribution with P = 0.40.
   a. Find the probability that number of successes is greater than 1,650.
   b. Find the probability that the number of successes is less than 1,530.
   c. Find the probability that the number of successes is between 1,550 and 1,650.
   d. With probability 0.09 the number of successes is less than how many?
   e. With probability 0.20 the number of successes is greater than how many?

Q8. A car rental company has determined that the probability a car will need service work in any given month is 0.2. The company has 900 cars.
   a. What is the probability that more than 200 cars will require service work in a particular month?
   b. What is the probability that fewer than 175 cars will need service work in a given month?

Q9. The tread life of a brand of tire can be represented by a normal distribution with mean 35,000 miles and standard deviation 4,000 miles. A sample of 100 of these tires is taken. What is the probability that more than 25 of them have tread lives of more than 38,000 miles?

Q10. Given an arrival process with λ = 5.0, what is the probability that an arrival occurs after t = 7 time units?

Q11. Times to gather preliminary information from arrivals at an outpatient clinic follow an exponential distribution with mean 15 minutes. Find the probability, for a randomly chosen arrival, that more than 18 minutes will be required.

Q12. A random variable X is normally distributed with mean 100 and variance 100, and a random variable Y is normally distributed with mean 200 and variance 400. The random variables have a correlation coefficient equal to - 0.5. Find the mean and variance of the random variable W = 5X + 4Y.

Q13. A random variable X is normally distributed with mean 500 and variance 100, and a random variable Y is normally distributed with mean 200 and variance 400. The random variables have a correlation coefficient equal to 0.5. Find the mean and variance of the random variable W = 5X – 4Y.

Q14. Shirley Johnson, portfolio manager, has asked you to analyze a newly acquired portfolio to determine its mean value and variability. The portfolio consists of 50 shares of Xylophone Music and 40 shares of Yankee Workshop. Analysis of past history indicates that the share price of Xylophone Music has a mean of 25 and a variance of 121. A similar analysis indicates that Yankee has a mean share price of 40 with a variance of 225. Your best evidence indicates that the share prices have a correlation of 0.5.
   a. Compute the mean and variance of the portfolio.
   b. Suppose that the correlation between share prices was actually – 0.5. Now what are the mean and variance of the portfolio?

Q15. The nation of Olecarl, located in the South Pacific, has asked you to analyze international trade patterns. You first discover that each year it exports 10 units and imports 10 units of wonderful stuff. The price of exports is a random variable with a mean of 100 and a variance of 100. The price of imports is a random variable with a mean of 90 and a variance of 400. In addition, you discover that the prices of imports and exports have a correlation of ρ = - 0.40. The prices of both exports and imports follow a normal probability density function. Define the balance of trade as the difference between the total revenue from exports and the total cost of imports.
   a. What are the mean and variance of the balance of trade?
   b. What is the probability that the balance of trade is negative?