HW Questions for chapter 12.

1. Solve the optimization problem for the following functions by using Lagrange-multiplier method & check the S.O.C.s for maximum or minimum by using bordered Hessian.
   a. $z = xy$ s.t. $x + 2y = 2$
   b. $z = x(y+4)$ s.t. $x + y = 8$
   c. $z = x - 3y - xy$ s.t. $x + y = 6$
   d. $z = 7 - y + x^2$ s.t. $x + y = 0$

2. Are the following functions quasiconcave? strictly so?
   First check graphically, then algebraically by using $\boxed{\text{Definition 1}}$ (without derivatives).
   Assume $x \geq 0$
   a. $f(x) = a$
   b. $f(x) = a + bx$ $(b > 0)$
   c. $f(x) = a + cx^2$ $(c < 0)$

3. Verify that a cubic function $z = ax^3 + bx^2 + cx + d$ is in general neither quasiconcave or quasiconvex.

4. Use $\boxed{\text{Definition 2}}$ to check a. $z = x^2$ $(x \geq 0)$ for quasiconcavity + quasi-convexity
   b. $z = xy$ $(x, y \geq 0)$

5. Use bordered determinants to check the following functions for quasiconcavity & quasiconvexity.
   a. $z = -x^2 - y^2$ $(x, y > 0)$
   b. $z = -(x+1)^2 - (y+2)^2$ $(x, y > 0)$

6. This is a HW question given in the class.
   Given $U = (x+2)(y+1)$ s.t. $P_x X + P_y Y = B$.
   a. Find x^* & y^* expressions
   b. Check 2nd order condition
   c. Derive $\frac{\partial x^*}{\partial B}$ & $\frac{\partial y^*}{\partial B}$.
   d. Derive $\frac{\partial x^*}{\partial P_x}$, $\frac{\partial y^*}{\partial P_x}$
   e. By setting $P_x = 4$, $P_y = 6$, $B = 130$, evaluate your answers a, b, c & d.

7. Determine whether the following functions are homogeneous. If so, of what degree?

a. $f(x,y) = \sqrt{xy}$

b. $f(x,y) = (x^2 - y^2)^{1/2}$

c. $f(x,y) = x^3 - xy + y^3$

d. $f(x,y) = 2x + y + 3\sqrt{xy}$

e. $f(x,y,w) = x^4 - 5yw^3$

8. Given the production function, $Q = AK^{\alpha}L^{\beta}$, show that:

a. $\alpha + \beta > 1$ implies increasing returns to scale

b. $\alpha + \beta < 1$ " decreasing "

7. Let output be a function of three inputs: $Q = AK^{a}L^{b}N^{c}$.

a. Is this function homogeneous? If so, of what degree?

b. Under what condition would there be constant returns to scale?

8. Let $H = e^{Q}$ where $Q = Aa^{\alpha}b^{\beta}$.

a. Confirm that H is a homothetic function

b. Show that the slope of H is constant for any given input ratio