Eco241 HW Questions for chapter 4

1. ① Rewrite the equation system in the matrix format $AX = d$.

$$Q_d = a - bP$$
$$Q_s = -c + dP$$
$$Q_d = Q_s$$

where $X = \begin{bmatrix} Q_d \\ Q_s \\ P \end{bmatrix}$

② Derive the expressions for the equilibrium $P$ and $Q$.

2. Rewrite the equation system in the matrix format $AX = d$

$$Q_1^d = a_0 + a_1 P_1 + a_2 P_2$$
$$Q_1^s = b_0 + b_1 P_1 + b_2 P_2$$
$$Q_1^d = Q_1^s$$
$$Q_2^d = \alpha_0 + \alpha_1 P_1 + \alpha_2 P_2$$
$$Q_2^s = \beta_0 + \beta_1 P_1 + \beta_2 P_2$$
$$Q_2^d = Q_2^s$$

where $X = \begin{bmatrix} Q_{d_1} \\ Q_{s_1} \\ Q_{d_2} \\ Q_{s_2} \\ P_1 \\ P_2 \end{bmatrix}$

3. Given $A = \begin{bmatrix} 2 & 8 \\ 3 & 0 \\ 5 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 0 \\ 3 & 8 \end{bmatrix}$ and $C = \begin{bmatrix} 7 & 2 \\ 6 & 3 \end{bmatrix}$.

(a) Is $AB$ defined? Calculate $AB$. Can you calculate $BA$? Why?

(b) Is $BC$ defined? Calculate $BC$. Is $CB$ defined? If so, calculate $CB$. Is it true that $BC = CB$?

4. Find the product matrices in the following

(a) $\begin{bmatrix} 0 & 1 & 0 \\ 3 & 0 & 4 \\ 2 & 3 & 0 \end{bmatrix}\begin{bmatrix} 8 & 0 \\ 0 & 1 \\ 3 & 5 \end{bmatrix}$ (b) $\begin{bmatrix} 3 & 2 & 0 \\ 4 & 2 & -7 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ (c) $\begin{bmatrix} a & b & c \end{bmatrix}\begin{bmatrix} 7 & 0 \\ 0 & 2 \\ 1 & 4 \end{bmatrix}$

5. Rewrite the following in $\Sigma$ notation:

(a) $\frac{1}{x} + \frac{1}{x^2} + \cdots + \frac{1}{x^n}$ $(x \neq 0)$

(b) $1 + \frac{1}{x} + \frac{1}{x^2} + \cdots + \frac{1}{x^n}$ $(x \neq 0)$

6. Given $w=\begin{bmatrix}3\\2\\16\end{bmatrix}$, $x=\begin{bmatrix}x_1\\x_2\end{bmatrix}$, $y=\begin{bmatrix}y_1\\y_2\end{bmatrix}$ and $z=\begin{bmatrix}z_1\\z_2\end{bmatrix}$

(a) Which of the following are defined:
$w'x$, $x'y'$, $xy'$, $y'y$, $zz'$, $yw'$, $x.y$ ?

(b) Find all the products that are defined.

7. Having bought $n$ items of marchandise at quantities $Q_1 \dots Q_n$ and Prices $P_1 \dots P_n$, how would you express the total cost of purchase in (a) $\sum$ notation and (b) vector notation?

8. In the 3-dimensional euclidean space, what is the distance between the following points?

(a) $(3, 2, 8)$ and $(0, -1, 5)$

(b) $(9, 0, 4)$ and $(2, 0, -4)$

9. Given $A=\begin{bmatrix}-1 & 8 & 7\\0 & -2 & 4\end{bmatrix}$, $b=\begin{bmatrix}9\\6\\0\end{bmatrix}$ and $x=\begin{bmatrix}x_1\\x_2\end{bmatrix}$:

(a) Calculate (a) $AI$ (b) $IA$ (c) $Ix$ (d) $x'I$
(e) $Ab$ (f) $AIb$ (g) $x'IA$ (h) $x'A$

10. Given $A=\begin{bmatrix}2 & 4\\-1 & 3\end{bmatrix}$, $B=\begin{bmatrix}3 & 8\\0 & 1\end{bmatrix}$ and $C=\begin{bmatrix}1 & 0 & 9\\6 & 1 & 1\end{bmatrix}$

Verify

(a) $(A+B)' = A'+B'$

(b) $(AC)' = C'A'$