ECO 241 HW Questions for Chapter 7.

1. Differentiate the following with respect to x.

(a) $(2-3x)(1+x)(x+2)$

(b) $(x^2+3)x^{-1}$

(c) $\dfrac{ax^2+b}{cx+d}$

(d) $5e^{-x}$

(e) $2\ln x$

(f) 5^x

(g) $(8x^3-5)^9$

(h) $(ax+b)^4$

(i) $\dfrac{2x-3y}{x+y}$

(j) $\dfrac{x^2-1}{xy}$

2. By using the definition of derivative (from chapter 7), prove that the derivative of $f(x)=3x^2$ is $f'(x)=6x$.

3. Given $y=-5x+20$

(a) Find its inverse function

(b) Find $\dfrac{dy}{dx}$ & $\dfrac{dx}{dy}$ & verify the inverse function rule.

4. Is the following function monotonic? If so, find $\dfrac{dx}{dy}$ by the inverse-function rule

$$y=x^3-4x^2+10x+100$$

5. Given the total-cost function,

$$C=Q^3-13Q^2+80Q+100$$

(a) find ① Average total cost function (ATC) ② Average variable cost fn. (AVC) ③ Marginal cost function (MC)

(b) find Q for ① break-even point & ② shut down point.

Total fixed cost (2)

6. Given $AVC = Q^2 - 10Q + 50$ and $TFC = 80$.

① Derive Total cost function

② Derive marginal cost function

7. Given the demand function $Q_d = 30 - 2\sqrt{P}$.

① Derive inverse demand function

② Derive total revenue function as a function of Q

③ Derive marginal revenue function " "

④ Derive average revenue function " "

⑤ Plot MR & AR curves.

8. Given $AR = 60 - 3Q$

① find total revenue function

② " marginal revenue function

③ Plot AR & MR functions

9. Given the production function $Q = 96 K^{0.3} L^{0.7}$,

① find the marginal product of labor (MP_L) and marginal product of capital (MP_K)

② Confirm the law of diminishing marginal productivity of labor & capital by computing the second derivatives of Q.

③ As K increases, what happen to MP_L?
" L " " MP_K?

④ Compute point elasticity of output with respect to labor, capital.
" "
Show your calculation.

10. If the utility function of an individual takes the form

$$U = U(x_1, x_2) = (x_1 + 2)^2 (x_2 + 3)^3$$

where U is total utility and x_1 and x_2 are the quantities of two commodities consumed.

(a) Find the marginal-utility function of each of the two commodities

(b) Find the value of the marginal utility of the 1st commodity when 3 units of each commodity are consumed.

11. Given the following market model, answer the questions

$$Q^d = a - bP$$
$$Q^s = -c + dP$$

(a) When a increases, will Q^* (equilibrium quantity) increase/decrease?

(b) b "

(c) c "

(d) d "

(e) Draw figures for cases (a)–(d) & confirm your findings

12. Given the following national income model,

$$Y = C + I_0 + G_0$$
$$C = \alpha + \beta(Y - T) \quad (\alpha > 0,\ 0 < \beta < 1)$$
$$T = \gamma + \delta Y \quad (\gamma > 0,\ 0 < \delta < 1)$$

(a) Consider the impact of $I_0 \uparrow$ on Y^*.

(b) " $\alpha \uparrow$ on Y^*

(c) " $\beta \uparrow$ "

(d) Derive the elasticity of national income w.r.t. δ. Interpret your result.

13. Given the following national income model, answer the questions (4)

$$Y = C + I_0 + G$$
$$C = 100 + 0.5(Y - T_0)$$
$$G = 0.2Y$$
$$I_0 = 1000$$
$$T_0 = 200$$

(a) List endogenous variables, exogenous variables + exogenous parameters.

(b) Compute the elasticity of national income w.r.t. g.

(c) Interpret the result in (b) by computing the values of g & Y^* after the 1% ↑ in g.

14. Use Jacobian determinants to test the existence of functional dependence between the functions below:

(a) $y_1 = 3x_1^2 + x_2$
$y_2 = 9x_1^4 + 6x_1^2(x_2 + 4) + x_2(x_2 + 8) + 12$

(b) $y_1 = 3x_1^2 + 2x_2^2$
$y_2 = 5x_1 + 1$