

ECO611 Homework Questions 1 <Due 11/10/2012 at 13:00pm>

1. For each of the following sets A and B, state whether $A \Rightarrow B$, $A \Leftarrow B$, or $A \Leftrightarrow B$. Do any of these sets satisfy the necessary and sufficient condition?

- (a) A = even numbered years, B = years in which the Summer Olympic Game are held.
- (b) A = a function is monotonic, B = a function is increasing.
- (c) A = a function is linear, B = a function is monotonic.
- (d) A = a function is one-to-one, B = a function has an inverse function.

2. Sketch the function $y = x^2 - 8x + 16$ over the interval $[1, 5]$. Draw a secant line on the function that connects the points $x_A = 1$ and $x_B = 3$.

- (a) If $x' = \lambda x_A + (1 - \lambda)x_B = 2$ is a point on the secant line, determine the value of $y' = \lambda f(x_A) + (1 - \lambda)f(x_B)$.
- (b) What is the slope of the secant line?
- (c) What can you say about this function's concavity or convexity?

3. Sketch the function $y = 8 + 10x - x^2$ over the domain $[0, 7]$.

- (a) Assume that $x_A = 1$, $x_B = 4$ and $\lambda = 0.4$. Using the formula $x' = \lambda x_A + (1 - \lambda)x_B$, determine the value of x' . What is the value of $f(x')$?
- (b) Calculate the value of $y' = \lambda f(x_A) + (1 - \lambda)f(x_B)$.
- (c) Prove that the above function is strictly concave by demonstrating $f(x') > y'$.

4. In applying mathematical methods to economic problems, economists utilize a broad menu of functions for the purpose of accurately specifying the properties of the economic variables in question. Choose the appropriate functional type from the following list of functions for that most accurately specifies the properties of each economic relationship described. Draw the general shape of each function as well.

- (i) $y = ax + b$; ($a < 0, b > 0$)
- (ii) $y = ax^2 + bx + c$; ($a > 0, b < 0, c > 0$)
- (iii) $y = \log_a(x - 1)$; ($a > 1$)
- (iv) $y = ax^3 + bx^2 + cx$; ($a > 0, b < 0, c > 0$)
- (v) $y = A \cdot x^a$; ($a < 0$)

- (a) A production function of one output and one input that exhibits increasing output with an increase in the input but diminishing marginal returns.
- (b) An indifference curve with two goods that exhibits a diminishing marginal rate of substitution (MRS).
- (c) An indifference curve with two goods that are substitutes and that exhibit a constant rate of substitution.
- (d) A marginal cost function of one output that exhibits diminishing marginal cost when output is small but increasing marginal cost when output gets larger.
- (e) A production function of one output with one input that exhibits increasing returns to scale when output is small and decreasing returns to scale when output is increasing.

5. Consider a function that relates tax revenues R, in billions of dollars, to the average tax rate t such that

$$R = 350t - 500t^2.$$

- (a) What tax rate(s) is consistent with raising tax revenues equal to \$60 billion?
- (b) What tax rate(s) is consistent with raising tax revenues equal to \$61.25 billion?